Basic Uncertainty Rules

Addition and subtraction:

$$\delta(x+y) = \delta x + \delta y \; ; \qquad \delta(x-y) = \delta x + \delta y$$

Multiplication and division:

$$\frac{\delta(xy)}{|xy|} = \frac{\delta x}{|x|} + \frac{\delta y}{|y|} \; ; \qquad \frac{\delta(x/y)}{|x/y|} = \frac{\delta x}{|x|} + \frac{\delta y}{|y|}$$

Multiplication/exponentiation by exact value:

$$\delta(cx) = |c|\delta x \; ; \qquad \frac{\delta(x^c)}{|x^c|} = |c|\frac{\delta x}{|x|} \qquad (\text{if } \delta c = 0)$$

Derivative rule:

$$\delta[f(x,y,\ldots)] = \left|\frac{\partial f}{\partial x}\right| \delta x + \left|\frac{\partial f}{\partial y}\right| \delta y + \ldots$$

Multiple measurements (accounting for statistical fluctuations):
Assume N independent measurements; best if $N \geq 5$.

$$x = \text{average of measured values} \; ; \qquad \delta x = \text{Range}/N = (\text{highest} - \text{lowest})/N$$

If δx from the above formula is less than the instrument uncertainty (obtained from the uncertainty table) use the instrument uncertainty instead for δx.

Slope of a graph (hand-plotted only):

$$\text{slope} = \Delta y/\Delta x = (y_2 - y_1)/(x_2 - x_1) \; ; \qquad \delta(\text{slope}) = \text{lc}_y/\Delta x$$

where (x_1, y_1) and (x_2, y_2) are two points taken from the best fit line (do *not* use points from the data table), and lc_y ("least count in y") is the y-increment represented by the smallest square on the graph.

Comparison of one quantity to zero: A quantity A is said to *agree with zero* (i.e., the experimental results are consistent with the statement that $A = 0$) if

$$|A| \leq 2\delta A$$

Comparison of two quantities to each other (the discrepency test): Two quantities A and B are said to *agree with each other* (i.e., the experimental results are consistent with the statement that $A = B$) if their difference $A - B$ agrees with zero, i.e.,

$$|A - B| \leq 2(\delta A + \delta B)$$

Correct rounding of final results:

Step 1: Round the uncertainty to two significant figures (if two are available — otherwise use one).

Step 2: Round the value itself to the same decimal place as the uncertainty.

If using scientific notation to represent the value itself, first convert the uncertainty to the same power of 10 as the value itself, and then factor out the power of 10. Apply the above two steps to the mantissa (the factor which multiplies the power of 10).

Improved Uncertainty Rules

Addition and subtraction:

$$\delta(x+y) = \sqrt{(\delta x)^2 + (\delta y)^2}\ ; \qquad \delta(x-y) = \sqrt{(\delta x)^2 + (\delta y)^2}$$

Multiplication and division:

$$\frac{\delta(xy)}{|xy|} = \sqrt{\left(\frac{\delta x}{|x|}\right)^2 + \left(\frac{\delta y}{|y|}\right)^2}\ ; \qquad \frac{\delta(x/y)}{|x/y|} = \sqrt{\left(\frac{\delta x}{|x|}\right)^2 + \left(\frac{\delta y}{|y|}\right)^2}$$

Multiplication/exponentiation by exact value:

$$\delta(cx) = |c|\delta x\ ; \qquad \frac{\delta(x^c)}{|x^c|} = |c|\frac{\delta x}{|x|} \qquad (\text{if } \delta c = 0)$$

Derivative rule:

$$\delta[f(x,y,\ldots)] = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\delta y)^2 + \ldots}$$

Multiple measurements (accounting for statistical fluctuations):
Assume N independent measurements; best if $N \geq 5$.

$$x = x_{\text{avg}} = \frac{\sum_{i=1}^N x_i}{N}\ ; \qquad \delta x = \sigma_N = \sqrt{\frac{\sum_{i=1}^N (x_i - x_{\text{avg}})^2}{N(N-1)}}$$

To account for the instrument uncertainty (iu), calculate the standard deviation (σ_N) as above and use

$$\delta x = \sqrt{\sigma_N^2 + (\text{iu})^2}$$

Slope/intercept of a graph:
Use linear regression (separate handout).

Comparison of one quantity to zero: A quantity A agrees with zero if

$$|A| \leq 2\delta A$$

Comparison of two quantities to each other (the discrepency test): Two quantities A and B agree with each other if

$$|A - B| \leq 2\sqrt{(\delta A)^2 + (\delta B)^2}$$